IPP-SR-5: Spacetime structure from Aristotle to Minkowski

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HT25

The course

1. Newton's laws
2. Galilean invariance
3. The Michelson-Morley experiment
4. Einstein's 1905 derivation of the Lorentz transformations
5. Spacetime structure
6. General covariance
7. Relativity and conventionality of simultaneity
8. Frame-dependent effects
9. The twin paradox
10. Dynamical and geometrical approaches to relativity
11. Presentism and relativity
12. Acceleration and redshift

Today

Minkowski's 1908 paper

Kleinian and Riemannian conceptions of geometry

Spacetime structure in Newtonian mechanics

Spacetime structure in special relativity

Further reflections on spacetime

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- What was Einstein's reaction?

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- What was Einstein's reaction? He accused Minkowski's work of being "superfluous learnedness" (Pais 1982).

The world-postulate

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- **Question:** Is this the origin of a Friedman-style 'geometrical approach' to physical theories? (Cf. lecture 1.)

Today's goal

Our goal for today is to spell out the move from dynamical symmetries to spacetime structure.

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Two conceptions of geometry

Kleinian conception: Geometry is characterised via the invariance groups of certain structures under coordinate transformations.

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Inertial frames and spacetime structure

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- Sometimes, people also think about the inertial frames as those frames which respect spacetime's 'inertial structure' in a certain way.
- Today, we will see how this goes, from the Kleinian perspective.

Kleinian approach summarised

- Specify the class of coordinate transformations which relate the inertial frames in the theory under consideration.

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- Identify the structures and quantities which are *invariant* under those transformations.

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- Specify the class of coordinate transformations which relate the inertial frames in the theory under consideration.
- Identify the structures and quantities which are *invariant* under those transformations.
- Regard these structures and quantities as picking out different kinds of spacetime.

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A hierarchy of structures

Aristotelian spacetime

$\downarrow$

Newtonian spacetime

$\downarrow$

Neo-Newtonian/Galilean spacetime

$\downarrow$

Maxwellian/Newton-Huygens spacetime

$\downarrow$

Leibnizian spacetime

$\downarrow$

Machian spacetime

A hierarchy of structures

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$\downarrow$

Newtonian spacetime

$\downarrow$

Neo-Newtonian/Galilean spacetime

$\downarrow$

Maxwellian/Newton-Huygens spacetime

$\downarrow$

Leibnizian spacetime

$\downarrow$

Machian spacetime

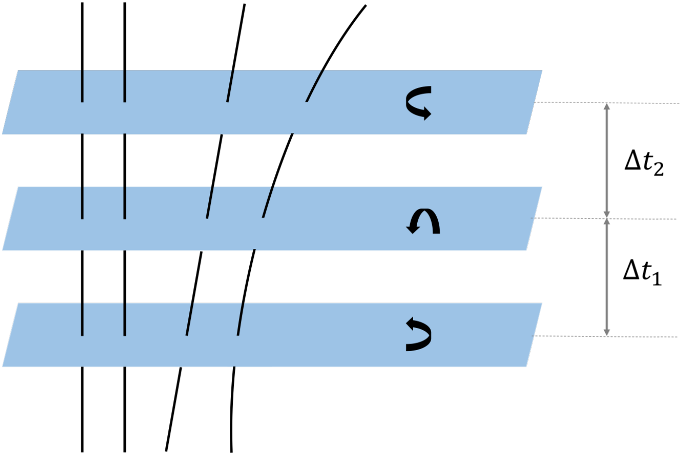
(Throughout the following, $\mathbf{R} \in SO(3)$ and any functions of t are smooth.)

Aristotelian spacetime

$$t \mapsto \pm t + \tau$$
$$\mathbf{x} \mapsto \mathbf{R}\mathbf{x}$$

In Aristotelian spacetime, there is:

1. A notion of spatial distance.
2. A notion of temporal distance.
3. A standard of rotation across time.
4. A notion of straightness of paths across time.
5. A preferred velocity.
6. A preferred point.



Δt_2
Δt_1

Newtonian spacetime

$$t \mapsto \pm t + \tau$$
$$\mathbf{x} \mapsto \mathbf{R}\mathbf{x} + \mathbf{a}$$

In Newtonian spacetime, there is:

1. A notion of spatial distance.
2. A notion of temporal distance.
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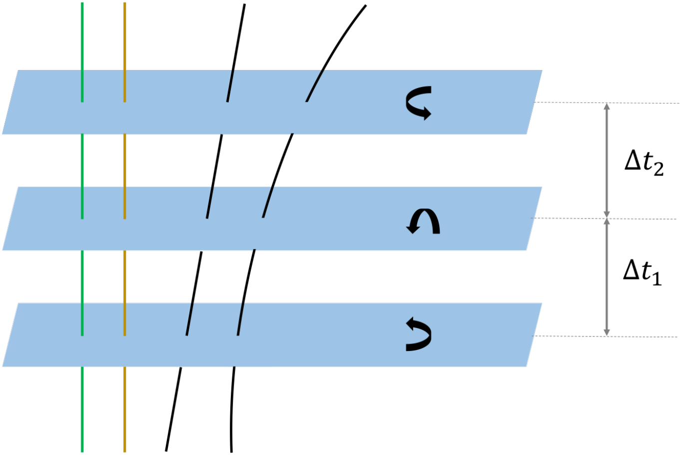

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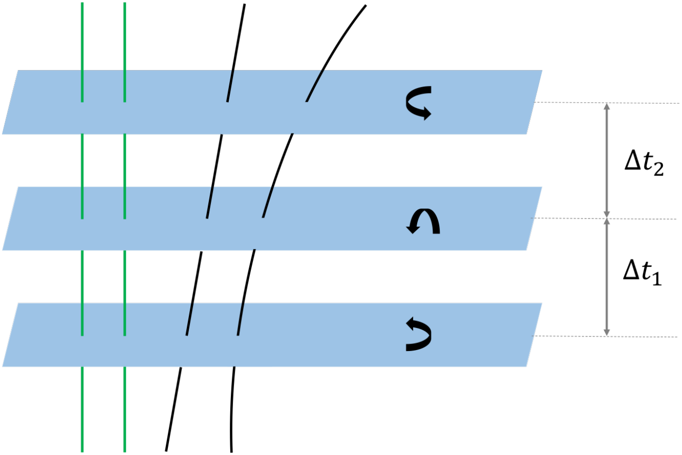


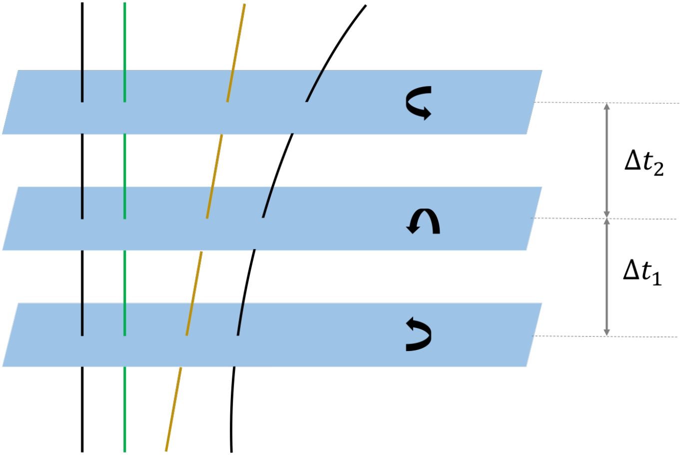
Δt_2
Δt_1

Neo-Newtonian/Galilean spacetime

$$t \mapsto \pm t + \tau$$
$$\mathbf{x} \mapsto \mathbf{R}\mathbf{x} + \mathbf{v}t + \mathbf{a}$$

In Neo-Newtonian/Galilean spacetime, there is:

1. A notion of spatial distance.
2. A notion of temporal distance.
3. A standard of rotation across time.
4. A notion of straightness of paths across time.
5. ~~A preferred velocity.~~
6. ~~A preferred point.~~



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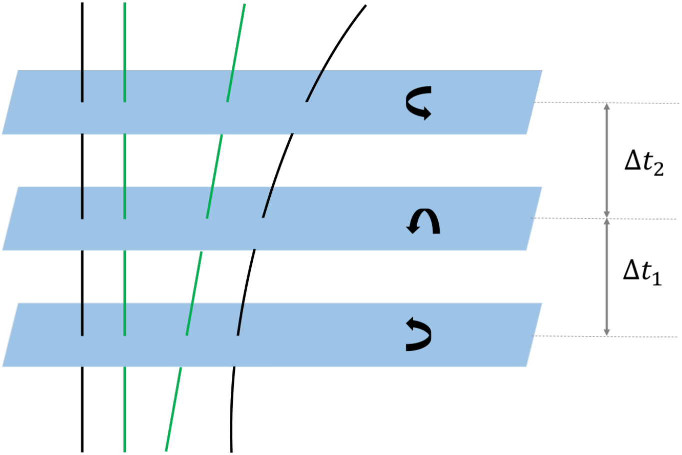

Δt_2
Δt_1

Maxwellian/Newton-Huygens spacetime

$$t \mapsto \pm t + \tau$$
$$\mathbf{x} \mapsto \mathbf{R}\mathbf{x} + \mathbf{a}(t)$$

In Maxwellian/Newton-Huygens spacetime, there is:

1. A notion of spatial distance.
2. A notion of temporal distance.
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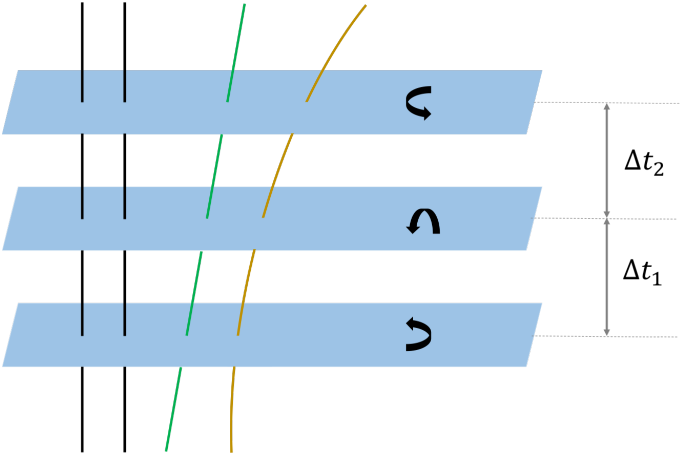
4. ~~A notion of straightness of paths across time.~~
5. ~~A preferred velocity.~~
6. ~~A preferred point.~~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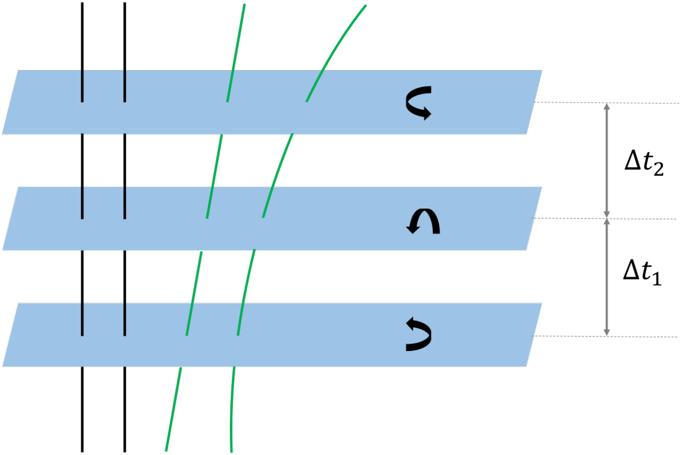
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Leibnizian spacetime

$$t \mapsto \pm t + \tau$$
$$\mathbf{x} \mapsto \mathbf{R}(t)\,\mathbf{x} + \mathbf{a}(t)$$

In Leibnizian spacetime, there is:

1. A notion of spatial distance.
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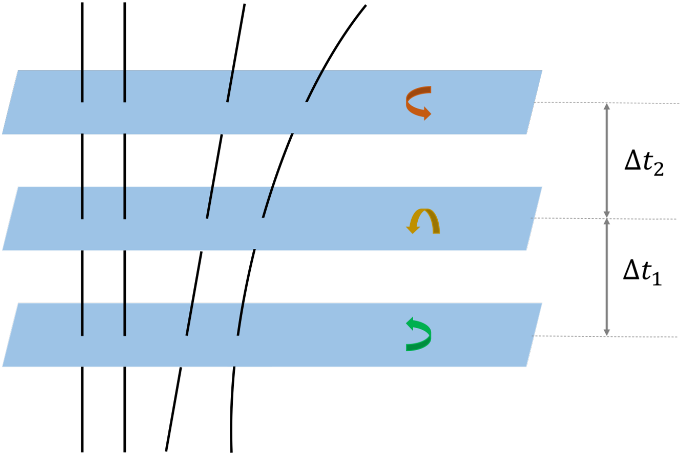

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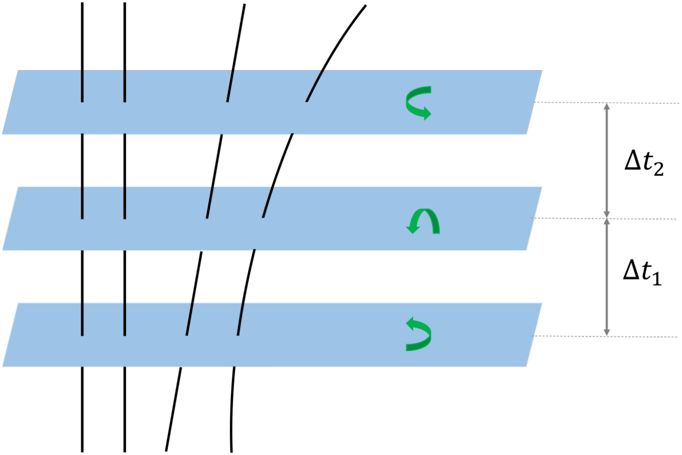
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Machian spacetime

$$t \mapsto f(t) \qquad\qquad (f \text{ monotonic})$$
$$\mathbf{x} \mapsto \mathbf{R}(t) + \mathbf{a}(t)$$

In Machian spacetime, there is:

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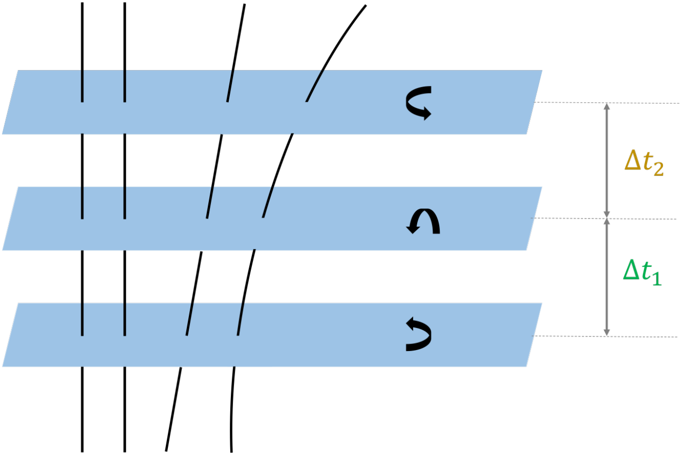

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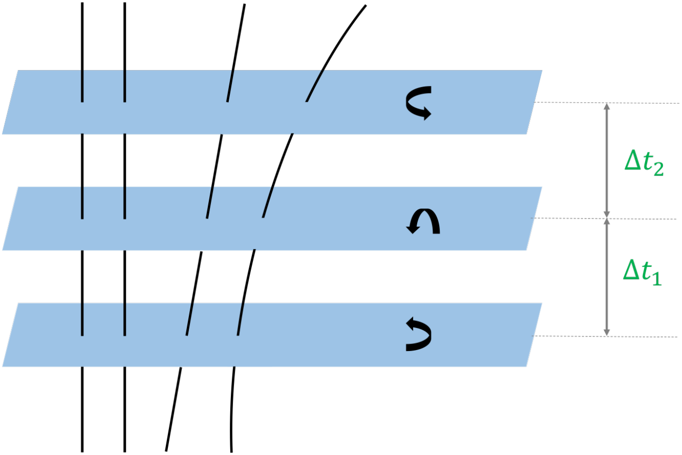

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Summary

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Question: How does the spacetime structure of special relativity compare with that of the spacetimes we have just seen?

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Aside: index notation

- Consider again the coordinate transformations associated with Galilean spacetime. So far, I've written these in vector notion, as

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- The equivalent expression in *index notation* would be

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- Note that all terms must have the same free indices, and the Einstein summation convention is used.
- By convention, we use Latin indices ($i, j, \ldots = 1, 2, 3$) for spatial indices, and Greek indices ($\mu, \nu, \ldots = 0, 1, 2, 3$) for space*time* indices.

Minkowski spacetime

$$x^{\mu} \mapsto \Lambda^{\mu}{}_{\nu} x^{\nu} + a^{\mu} \qquad (\Lambda^{\mu}{}_{\nu} \in SO(1,3))$$

In Minkowski spacetime, there is:

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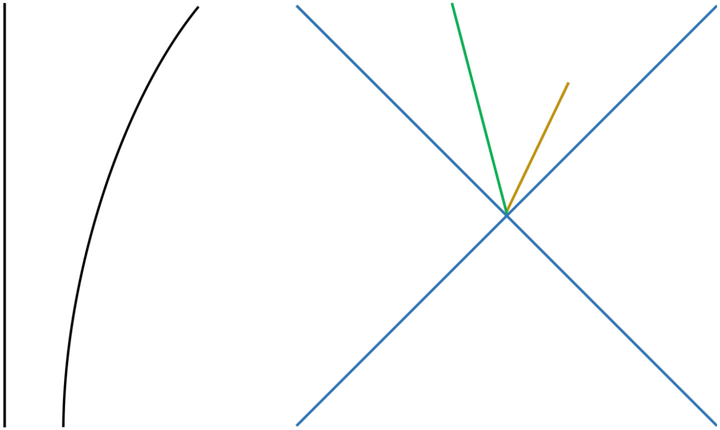
7. A notion of *spacetime* distance.



The interval

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$$I = -c^2 dt^2 + dx^2 + dy^2 + dz^2.$$

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$$I = -c^2 dt^2 + dx^2 + dy^2 + dz^2.$$
- I is preserved in all inertial frames in special relativity—i.e., in all frames related by Poincaré transformations.

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- But these are the transformations associated with Galilean spacetime, as we have seen above.
- It is natural, therefore, to regard Newtonian mechanics as being *set in Galilean spacetime*.

Earman's adequacy conditions

In (Earman 1989, ch. 3), Earman makes it a very general principle that the spacetime and dynamical symmetries of a theory should match, by laying down two conditions:

SP1: Any dynamical symmetry of T is a spacetime symmetry of T.

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(Some have gone further, by saying that these principles are *analytically true*—see e.g. (Myrvold 2017).)

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- With hindsight, Newton violated this requirement: Newtonian physics can be formulated in (merely) *Galilean* spacetime, not *Newtonian* spacetime (as Newton maintained). Occam's razor thus advises against postulating a standard of absolute rest in addition.

Correct spacetime setting for Newtonian mechanics

If we follow the methodology of moving from Newtonian to Galilean spacetime as the correct spacetime setting for Newtonian mechanics, then (it seems) the discovery of *further* invariances of the Newtonian laws would similarly motivate moving to a different spacetime setting again, with even less structure than Galilean spacetime.

Newton's 'Corollary VI'

- Consider Newton's 'Corollary VI' in the *Principia*:

  > *If bodies moved in any manner among themselves are urged, in the direction of parallel lines by equal accelerative forces, they will all continue to move among themselves, after the same manner as if they had not been urged by those forces.* (Cajori 1934, p. 21)

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- This is an ongoing matter of some controversy—see (Knox 2013) and (Wallace 2020) for further discussion.

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- If we impose extra structure on Minkowski spacetime (namely, again, a standard of rest), we can (again, perhaps surprisingly) recover Newtonian spacetime.
- So...:

  There is a precise sense in which Newtonian spacetime has more structure than both Galilean spacetime and Minkowski spacetime. But in this same sense, Galilean and Minkowski spacetime have incomparable amounts of structure; neither spacetime has less structure than the other. The progression towards a less structured spacetime therefore does not continue into the relativistic setting. (Barrett 2015, p. 37)

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1. Distinguished between Kleinian and Riemannian conceptions of geometry.
2. Witnessed the tower of classical spacetime structures.
3. Compared these classical spacetime structures with the structure of Minkowski spacetime.
4. Discussed the correct spacetime setting for Newtonian mechanics.

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